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The Elliptical Wing in-or-out of Ground Effect

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INTRODUCTION

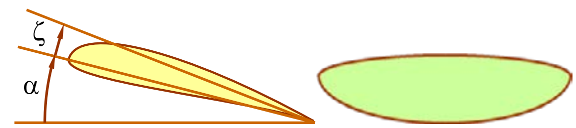
Even those of us with an extensive aerodynamic library may be surprised to learn that the well-known elliptical wing develops significantly lift, and significantly greater drag, than those advertised in the literature. These elliptical wing "shortcomings" (literature shortcomings) were perhaps first pointed out by Sighard Hoerner in his well-known book "Fluid Dynamic Drag." Herein we substantiate Hoerner's observations, and then extend our study of the elliptical wing to include the shape of its "planform wake" and the effects of ground proximity on induced drag.

We introduce a practical, two-dimensional approximation of wake rollup as seen in the plan view, then applied toward a top-level look at the formation flight of pelicans. Finally, we turn to ground effect, re-inventing "from scratch" the largely undocumented method used by Wieselsberger and Pohlhausen to compute the reduction of induced drag with the elliptical wing near the ground.

1.0 ELLIPTICAL WING LIFT AND DRAG IN FREE AIR

A wing kept well away from the ground is said to fly in "free air." Test data thereof from three sources, diverse in both time and space, are shown in Figure 1.0-1 for elliptical wings of aspect ratio 5.0, 6.0, and 6.67. The data collapses to a single curve falling 10% below the well-known lifting-line prediction. In the figure, we have normalized the lift coefficient (c_L) as a "group" which, given perfect agreement with theory, would reside precisely on the plot "diagonal." This lift group includes representative section efficiency (η) and wing aspect ratio (A). Wing angle of attack and section zero-lift *inclination* are designated (α, ζ), respectively.

Whereas we are indebted to Ludwig Prandtl and his young associate, Max Munk, for the *lifting-line theory*, we are also indebted to Klaus Krienes and Robert T. Jones for their more thorough analyses known as *lifting-surface theory*. The "efficiency" of an elliptical wing, in terms of the ratio of its lift via lifting-surface theory to that via lifting-line theory, is shown in Figure 1.0-2. Only for infinite aspect ratio do the two theories yield the same lift. Notice that the empirical formula added to Figure 1.0-2 representing elliptical wing "efficiency" predicts well the "lift shortfall" at aspect ratio (A=5) in Figure 1.0-1.



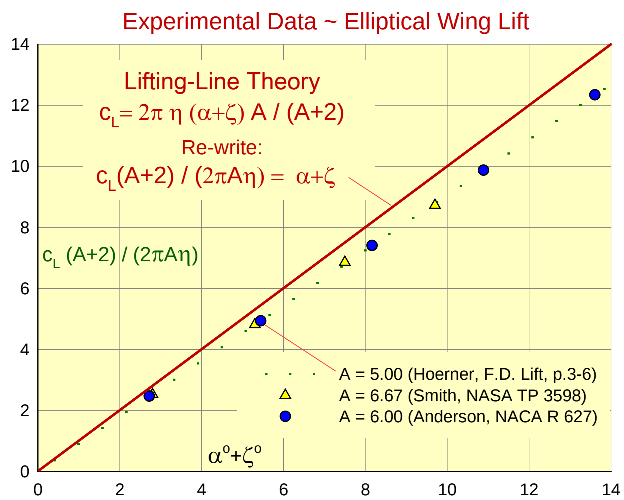


Figure 1.0-1 Elliptical Wing Lift Theory and Data

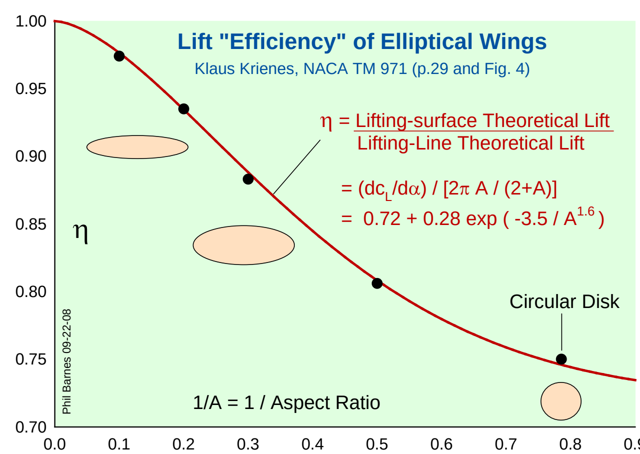


Figure 1.0-2 Elliptical Wing Lift via Lifting Surface Theory

In Figure 1.0-3 (next page), we characterize a "vortex drag group" for the test data which, given perfect agreement with theory, would lie on the plot diagonal. Again, we see a 10% reduction of performance. Yet notwithstanding the limitations of the lifting-line theory, it remains a powerful method which we will put to good use in our numerical analysis of induced drag later herein.

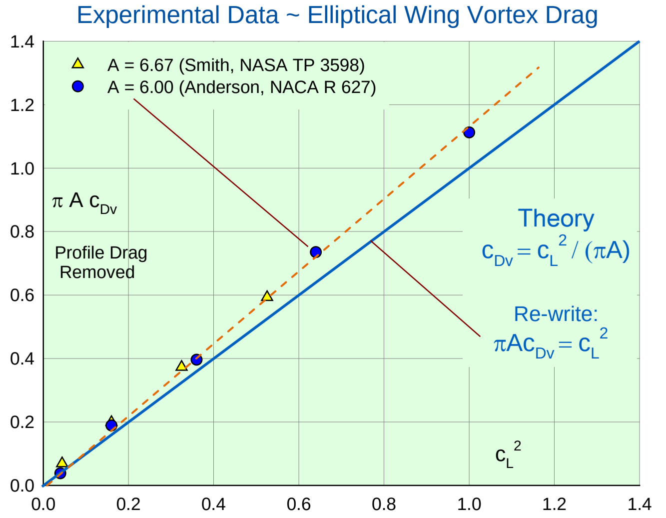


Figure 1.0-3 Elliptical Wing Vortex Drag ~ Theory and Data

2.0 WAKE ROLLUP

In this section take advantage of the classic work of Betz, Kaden, and Cone to provide an approximate characterization of the rate at which the wake of a wing rolls up. Here, our primary application is a top-level investigation of ground effect, where we ask: "In the formation flight of pelicans, is the wake from the lead bird essentially rolled up by the time it reaches the second bird?" To most expeditiously answer this question we limit ourselves to the appearance of the wake in the plan view. Based on the referenced papers, we propose the two-dimensional mathematical model of Figure 2.0-1 to approximate the shape of the "wake planform."

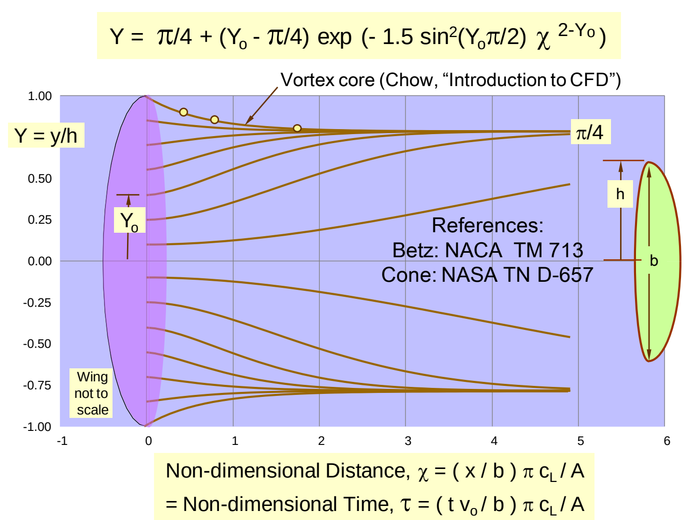


Figure 2.0-1 Elliptical Wing Vortex Drag ~ Theory and Data

Notice that the wake ultimately rolls up, in accordance with the theory of Betz, into a horseshoe vortex having a vortex-to-wing span ratio of (π/4). Note also that both the non-dimensional distance and non-dimensional time are proportional to the lift coefficient and inversely proportional to the aspect ratio, whereby the wake rolls up relatively fast for a low-aspect-ratio wing at high lift.

Now, to apply our model to the study of the formation flight of pelicans, we show various wings and their wakes, all drawn to scale, in Figure 2.0-2. If we assume that pelicans in formation are spaced longitudinally within one or two wingspans, we conclude from our analysis that those pelicans immediately following the lead pelican will experience very little wake-rollup effect. Figure 2.0-3 shows pelicans in free-air formation.

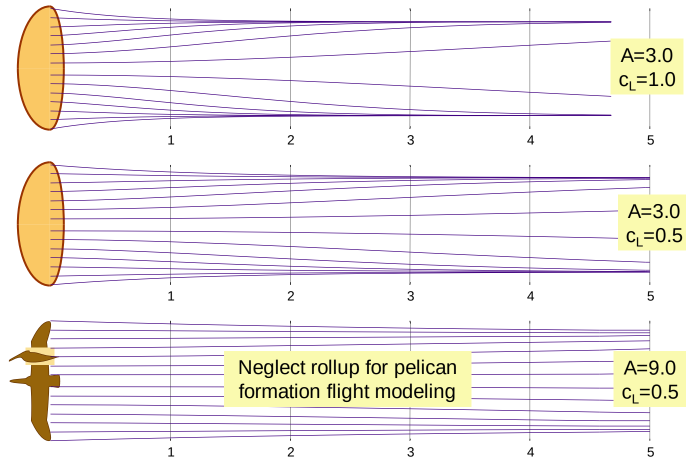


Figure 2.0-2 Wake Planform Studies

Figure 2.0-3 Pelicans in Formation

3.0 GROUND EFFECT

We are indebted to Albert Betz for his *method of images,* whereby an aircraft in ground effect can be imagined to have an inverted twin beneath the ground (or water), with all bound and trailing vortices rotating opposite to their counterparts above the ground. As a result of the mutual interaction of all the vortices, the image aircraft induces both *upwash* and *streamwash* (the latter negative) upon its upper twin, resulting in increased lift, reduced airspeed, and a negative increment in pitching moment (nose-down effect). These phenomena are illustrated in Figure 3.0-1 using a horseshoe vortex representation.

The classic analysis of induced drag in ground effect was carried out by C. Wieselsberger (NACA TM 77) with the aid of his assistant, K. Pohlhausen, and the suggestion of Ludwig Prandtl to assume that the loading remains elliptical as ground proximity increases (we show later herein that the wing tips are progressively unloaded, but Prandtl's assumption introduces only a minor error). As happens often in scientific papers, the paper author delegated the actual computation to an assistant.

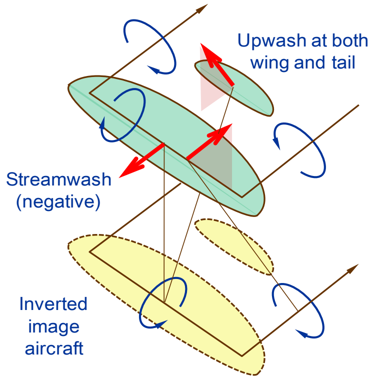


Figure 3.0-1 Vortex Interaction in Ground Effect

Herein, we re-invent the calculations lost to Mr. Pohlhausen's notebook, while following the top-level outline of the referenced paper. We study an isolated elliptical wing, integrating from wingtip-to-wingtip the upwash due to the mirror image. Before conducting the analysis, we can expect lift to increase and drag to decrease as the wing approaches the ground. But at this point we ask: "Does aspect ratio influence the reduction of induced drag for a given lift coefficient?"

Although the details of the method are found in the Appendix for the benefit of the interested reader, we point out here the highlights of the method and its results. First, the analysis normalizes the local upwash velocity as a ratio (υ=u/v_o) to freestream velocity. This upwash angle is further normalized as a ratio with the free-air downwash angle (δ_o) which, for elliptical loading, is given by [$c_L/(\pi A)$]. When the normalized upwash is integrated (Appendix) and then plotted versus the non-dimensional spanwise coordinate (Y=y/h) we obtain the family of curves shown in Figure 3.0-2.

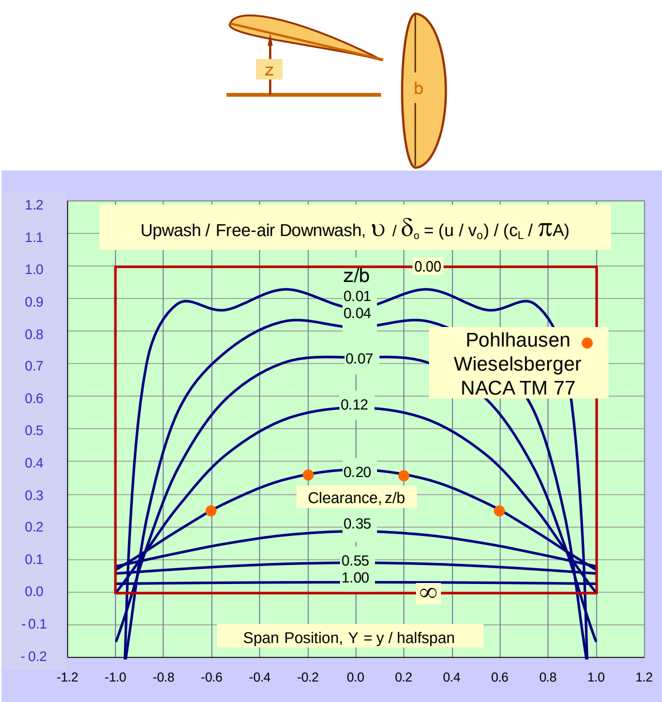
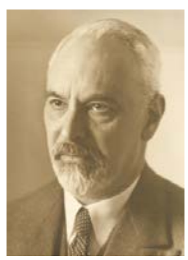


Figure 3.0-2 Integrated Normalized Upwash in Ground Effect

Here, an interesting characteristic is a "bounding box" with a lower limit representing free air (no upwash) and an upper limit representing lifting-line contact with the ground and total cancellation of free-air downwash, with induced drag vanishing in the limit. The curves in between represent various non-dimensional elevations (z/b) expressed in "wingspans." These calculations, performed with the aid of a computer, agree well with the apparently laborious hand calculations carried out by Pohlhausen. We point out that our sign convention differs from that of Pohlhausen by treating both upwash and downwash as positive in their respective directions.

Key results of the analysis include (a) wingtips are unloaded as [z/b] decreases and (b) the induced drag reduction is independent of aspect ratio. This last result is made more clear by Figure 3.0-3, showing the effect of ground proximity on induced drag. Our numerical method agrees not only with the calculations of Wieselsberger and Pohlhausen, but also with test data representing both elliptical and rectangular wings. Albert Betz's *method of images*, and Prandtl's *elliptically-loaded wing* concept have once again been validated.

Albert Betz



Ludwig Prandtl

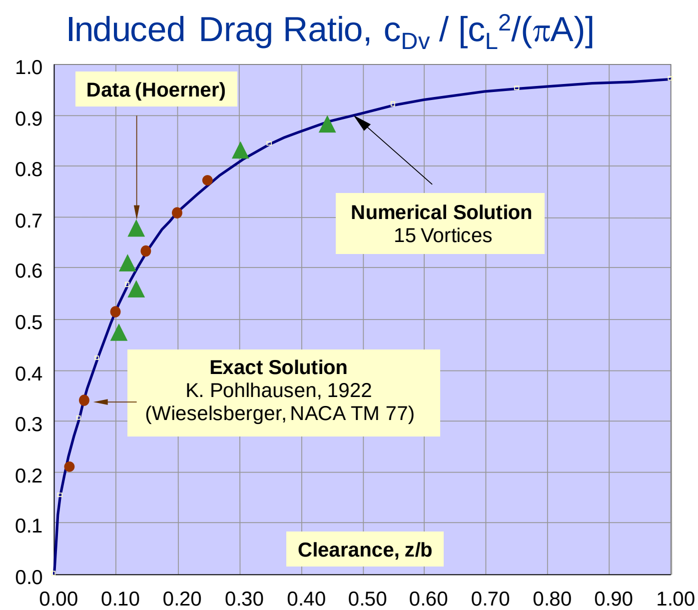
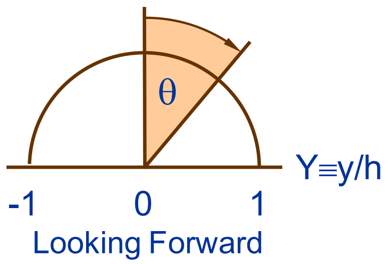


Figure 3.0-3 Induced Drag Ratio in Ground Effect

APPENDIX Numerical Integration of Upwash in Ground Effect

"Glauert Coordinate" $Y \equiv \sin\theta$



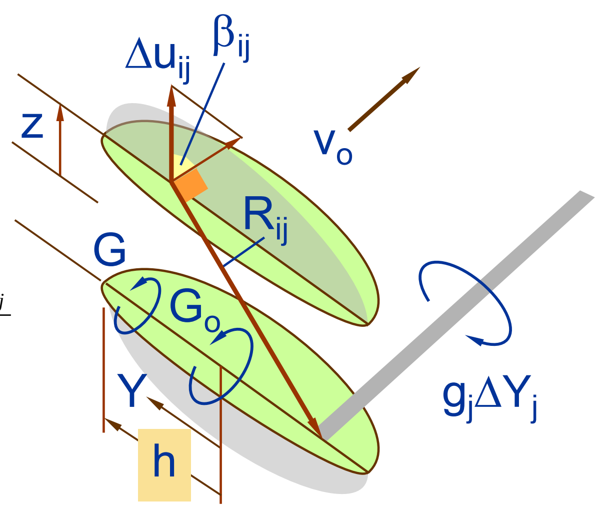
$Y \equiv y/h$; $R \equiv r/h$; $Z \equiv z/h$

$G \equiv \Gamma / (v_o h) = G_o \cos\theta$

$g \equiv dG / dY = - G_o \tan\theta$

Numerically Integrate $F_{ij}(\theta)$

$-\pi/2 < \theta < \pi/2$ $\{i \neq j\}$, where:

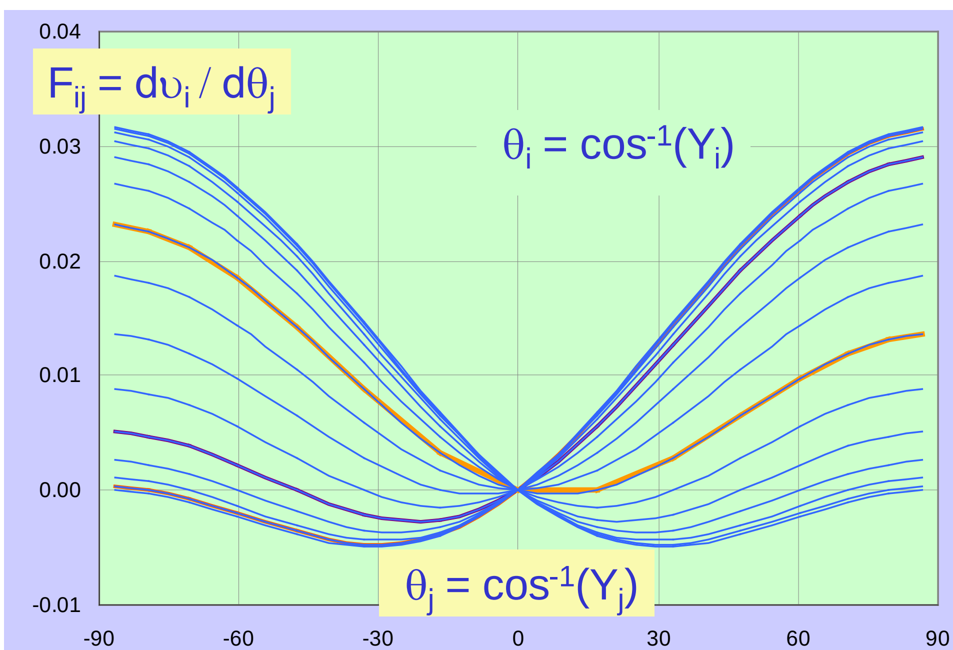


$$\upsilon_i \equiv u_i / v_o = \sum_j \Delta u_{ij} / v_o = \sum \frac{g_j \, \Delta Y_j}{4 \pi R_{ij}} \cos\beta_{ij}$$

$$\boxed{\frac{\upsilon_i}{c_L/(\pi A)}} = \frac{\upsilon_i}{G_o/4} = \sum \frac{-\tan\theta_j \cos\theta_j \, \Delta\theta_j \cos\beta_{ij}}{\pi R_{ij}}$$

$$= \sum F_{ij} \, \Delta\theta_j \qquad \text{where:} \quad \cos\beta_{ij} = \frac{\sin\theta_i - \sin\theta_j}{R_{ij}}$$

$$\text{and where:} \quad R_{ij} = \sqrt{(\sin\theta_i - \sin\theta_j)^2 + 4Z^2}$$



Upwash Integrand versus Span Position in Terms of the Glauert Coordinate (θ)